REDUCED ORDER GUIDANCE METHODS AND PROBABILISTIC TECHNIQUES IN ADDRESSING MISSION UNCERTAINTY

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Abstract

Recognizing that vehicle synthesis fulfills the role of integrator of the mutually interacting disciplines, difficulties persist in intelligently implementing disciplinary analysis into this synthesis process. This paper develops and describes analytical and statistical approximation techniques used to create design-oriented analyses which are implementable in the process. Specifically, techniques related to the vehicle guidance discipline are examined. The ultimate goal is to investigate the economic viability of an aerospace system in the face of uncertainty at the system and discipline design levels. The notion of a "design mission" as a requirement is replaced by a modeling of mission variability, since future aircraft will likely fly a variety of missions. Aircraft guidance laws are key components in the mission analysis portion of an aircraft sizing code, and thus they must be included in the investigation. Through the use of statistical modeling techniques, a link between mission uncertainty, optimal guidance, wing planform, and economic objectives is obtained. This linkage allows for the investigation of guidance and mission effects on such quantities as gross weight and ticket price (on a per mile basis). Further, the resulting solutions are robust since they are obtained by choosing control parameters which maximize the probability of meeting a target while simultaneously assuring that appropriate constraints (which are also probabilistic) are met.

Introduction

The consideration of economic uncertainty on the aircraft synthesis process is the focus of recent research as a means to generate robust solutions.[1,2] Building on the insights of this work, this paper expands the types of uncertainty considered to include variability related to the vehicle guidance discipline as well as to mission requirements. In general, defining requirements is a key first step to a successful design process. In fact, requirements definition is a key element of the Integrated Product and Process Development (IPPD)/Concurrent Engineering (CE) concept which has emerged recently as a viable means of implementing concurrent engineering practices in aerospace systems design.[3] Designing aircraft in an IPPD/CE framework is viewed as designing with a focus on affordability, which implies an understanding of how various discipline, mission, design, and economic variables affect the feasibility ("can it be built") and viability ("should it be built") of an aircraft. It is in *evaluating* feasibility and viability that one quickly realizes the non-deterministic nature of the design problem. This is because parameters traditionally treated as fixed assumptions are in reality distributions around a most likely value. For example, deterministic methods proceed "assuming a fuel price of $1/gallon" whereas probabilistic methods proceed "assuming a range of possible fuel prices represented by a probability distribution". Probabilistic design methods have, until recently, been discipline specific. In recent years, research in the controls area has had its emphasis on robustness, focusing on the conflicting

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goals of performance and disturbance rejection as well as how best to model uncertainty in controller design.[4,5] A variety of stochastic design methods are being applied in the structures and reliability disciplines as well.[6] In the discussions that follow here, robustness and uncertainty modeling are approached with system and discipline level objectives in mind. The approach used is part of a Robust Design Simulation (RDS) method currently under development at Georgia Tech's Aerospace System Design Laboratory (ASDL).[7]

The proper place to assess the effect of these system and discipline uncertainties is in the aircraft sizing and synthesis process. A key part of this process is the guidance law to be used (which defines the trajectories flown along the defined mission). Trajectory optimization problems (and the related optimal guidance problem) are usually approached from the "given a vehicle and a mission, this is its optimal trajectory" point of view. This after-the-fact (i.e. after design decisions, such as configuration parameter selection, have been made) analysis then precludes optimal guidance as a design driver in a multidisciplinary design process. One reason for this situation is that the modeling complexity generally employed for these flight mechanics problems requires intensive numerical effort for their solution. This is usually the case whether indirect methods (involving discretization of the state and control parameters) or direct methods (generally necessitating the solution of a Two-Point Boundary Value Problem (TPBVP)) are employed in solving the optimal control problem. In light of these issues, an efficient and, if possible, analytical approach to guidance optimization is needed for highly iterative, complex multidisciplinary problems.

This paper explores how using multiple-time scale analysis and a Singular Perturbation (SP) formulation can fulfill this need. SP techniques offer analytical clarity as well as computational efficiency over more exact indirect, numerical approaches or traditional full order direct methods (involving TPBVPs) in solving certain flight mechanics problems.

Reduced order guidance techniques have been under development since the late 1960's, with a key motivation being performance analysis and real time, onboard implementation. The same characteristics which make these techniques attractive for real time implementation (e.g. computational efficiency, analytical insight, flexibility) also make them desirable for application to multidisciplinary analysis and design problems. The sizing and synthesis code used in this study, the Flight Optimization System (FLOPS)[8], has had reduced order guidance schemes within it for several years, based on References 9 and 10. Fortunately, the infrastructure in the code allows for the implementation of alternate, potentially more advanced, schemes for investigation. This paper demonstrates that using SP theory, one can compute the same guidance laws currently in FLOPS, but with a more clear understanding of what is really happening in the dynamic model. Further, this initial demonstration is left open-ended to make room for future extensions of the theory and its use. SP-based guidance law development has been shown to allow the solution of problems which were not easily solvable using standard optimal control theory.[11] A good review article on flight mechanics application of the techniques can be found in Reference 12.

Finally, it seems clear that finding an analytical relationship between wing geometric characteristics, mission parameters, optimal guidance solutions, and aircraft economics is not feasible. Thus, experimental techniques are investigated with the hope that they will allow the approximation of these unknown relationships via statistical analysis of a set of properly generated data. In terms of the guidance solutions themselves, experimental techniques are unnecessary since the afore mentioned reduced-ordered analytical modeling is adequate for performance studies. A Design of Experiments (DOE)/Response Surface Methodology (RSM) approach is used at several stages of the design optimization process. Reference 13 documents how a set of parametric response surface equations (RSEs) which predict the aerodynamic performance of a High Speed Civil Transport (HSCT) were generated. These RSEs are used within the sizing routines of FLOPS in the present study to generate a new set of RSEs which provide the desired relationship (albeit approximate) between mission, guidance, planform, and economic considerations. Finally, as part of the RDS process, the DOE/RSM technique is used a third time to complete the process by using the approximate relationships to search for robust solutions via probability distributions for the $/RPM as well as performance constraints. Caveats in the application of these statistical techniques are discussed as appropriate.

Problem Formulation

The problem under consideration is to determine the values of a set of design variables which result in an HSCT which is robust to modeled uncertainty and satisfies imposed constraints. In the RDS method to be employed, the metric used to measure this robustness is the probability of a selected response meeting a specified target. For the particular application of this study, the selected response is the average yield per Revenue Passenger Mile ($/RPM).

This metric implicitly represents the ticket price, on a per mile basis, that an airline must charge in order to achieve a specified return on investment (ROI) while accounting for a required ROI for the manufacturer of the aircraft. Given this overall objective, the design variables and assumptions made in the formulation are described next. Then the aircraft sizing and synthesis process to be used will be presented, identifying how the climb optimization analyses are incorporated.

Classification and Description of Design Variables

Fully parameterizing an aircraft can require a set of design variables which is quite large, especially for an unconventional vehicle such as an HSCT. Figure 1 shows just the planform design variables involved in describing a kinked wing typical of supersonic transport designs. The DOE/RSE technique provides a way of reducing this set of variables to a more manageable number. However, even with the computational advantages brought by DOE, an excessive number of design variables can make the RSE generation expensive and/or difficult. Thus, an appropriate first step in defining a design space for further investigation is to conduct a screening test. A screening test is designed to identify the subset of design variables which contribute most to a given response. These are the variables for which the response has the highest sensitivity. The design variables which are to comprise the RSE capturing the essence of FLOPS must be the ones which have the most influence on the responses to be tracked. A screening process for the planform variables (Figure 1) has already been accomplished in reference 13. Based on those results and the goals of the current study, ten design variables are chosen for investigation. However, before summarizing the ten, a distinction needs to be made between the two types of design variables considered, control and random variables

Control variables are those which the designer can select to optimize an objective function. Typical examples of control variables are geometric quantities such as seen in Figure 1. Random (or noise) variables, on the other hand, represent parameters which cannot be freely chosen by the designer. Thus, to the designer, they represent uncertainty. The introduction of these random variables into the design approach implies the presence of stochastic processes; thus, resulting responses will necessarily be stochastic as well.

Of the ten HSCT design variables studied here, five are control variables and five can have random values (Table I). These variables represent aerodynamic, sizing, mission, and economic quantities. Minimum and maximum values for each define the extent of the design space. Validity of results are not guaranteed if regression equations are evaluated outside the ranges defined by these values.

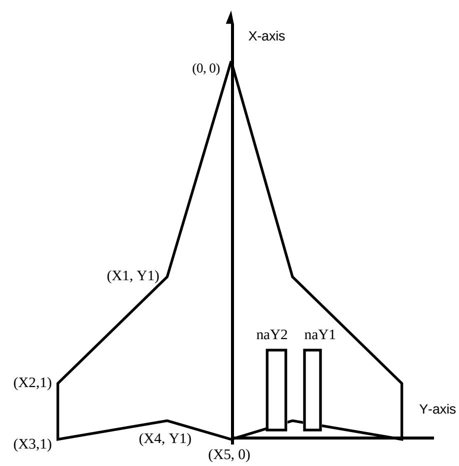

Figure 1: Parametric wing planform definition (values normalized by semi-span)

Table I: HSCT Problem Design Variables and Classifications

Type	*Variable*	*Group*	*Symbol*	*Minimum*	*Maximum*
Control	Wing Kink X-location	Aerodynamic	X1	1.54	1.69
	Wing Kink Y-location	Aerodynamic	Y1	0.44	0.58
	Wing Ref. Area	Sizing	Sref	8500 sq. ft.	9500 sq. ft.
	Thrust/Weight	Sizing	TWR	0.28	0.32
	Mission Range	Mission	DESRNG	5000 nm	6000 nm
Random	%Subsonic- Leg1	Mission	SUBL1	0 %	15 %
	%Subsonic- Leg2	Mission	SUBL2	0 %	15 %
	Climb Optimization	Mission	CLIMB	0 (min time)	1 (min fuel)
	Fuel Cost	Economic	COFL	0.55 $/gal	1.1 $/gal
	Economic Range	Economic	EcRNG	3000 nm	5000 nm

Figure 2: Possible HSCT Mission Profiles

fuel flow tables. This vehicle is then "flown" along a designated mission based on some guidance laws for climb, cruise, descent, etc. If, at the end of the mission, the fuel available (determined from volume considerations) is equal within some tolerance to the fuel required (fuel used to fly the mission plus reserve fuel), the aircraft is said to be sized. If not, an iteration takes place by increasing/decreasing the fuel available as appropriate and re-flying the mission. Once converged, the main outputs include gross weight, fuel weight, and values for any number of performance constraints. This algorithm forms the core of most all synthesis codes, including FLOPS.

For this study, the vehicle aerodynamics used during synthesis are represented by a set of second order polynomial response surface equations (RSEs) previously generated and reported in reference 13. More specifically, these RSEs predict the components of the total drag coefficient as a function of wing planform and airfoil variables as well as the wing reference area. These RSEs have been integrated into FLOPS so that total drag can be calculated at any flight condition and lift coefficient. The parametric wing definition on which the equations are based is shown in Figure 1. The reasons aerodynamic RSEs are needed as opposed to the built in capability within FLOPS are well documented in the reference. One key reason is the lack of accuracy in the aerodynamic prediction routines within most synthesis codes when applied to unconventional aircraft, such as an HSCT.

Constraints are also tracked in the experiment, including key quantities which determine the certification feasibility of an HSCT. These are called external constraints and include approach speed, takeoff field length, and landing field length. A sizing run may converge without these constraints being met. Internal constraints are those associated with the mission that are required to be met within a converged solution (see Table II).

Table II: Internal and External Constraints

External	
Approach Speed	154 kts.
Takeoff Field Len.	11,000 ft.
Landing Field Len.	10,500 ft.
Internal	
Dynamic Pressure	2000 psf
Maximum Altitude	Subsonic: 40,000 ft. Supersonic: 70,000 ft.
Cruise Mach Number	Subsonic: 0.85 Supersonic: 2.4

FLOPS also has a module which predicts takeoff noise; thus, noise constraints can be investigated as well. However, determining designs optimized for noise requires a more detailed modeling than used here, especially in the terminal areas of the mission. However, the method presented here can be utilized for finding robust solutions for specific mission segments as well.

Now that the problem to be addressed has been defined and the number and type of design variables to be considered has been discussed, the method of solution is presented. This presentation is then followed by implementation of the approach on the HSCT example.

Guidance Solution Approach

Based on the description of the sizing process above, it is evident that a key element of subsequent sizing solutions are the guidance laws which dictate specifically how the mission is to be flown. As these laws change (for example with changing the climb optimization factor), so does the fuel consumption, total fuel weight, gross weight, operating cost, etc. all the way to $/RPM. This interdependence necessitates an understanding of the guidance techniques currently used in FLOPS and the methods are available to improve upon them.

Aircraft dynamic models range from nonlinear rigid body equations to point mass, one state variable models (such as the Energy State Approximation, ESA). In between are models of varying complexity and usefulness. Numerous results over the past decade have shown that point mass dynamic models are sufficient for performance studies.[12] Within the realm of point mass models, there are numerous levels of fidelity possible. For instance, three dimensional models can be utilized in this setting. However, for performance analysis of transport aircraft, it suffices to consider only motion in the vertical plane. Finally, reduced order models are appropriate for the level of mission analysis (and the level of the other disciplinary modules) in FLOPS addressed in this paper.

Currently, there are two approaches for climb optimization are present in FLOPS. After the first (and simplest) of the two is described, the second will actually be derived via the SP technique. The simplest of the two climb solutions in FLOPS is based on the ESA.[10] The energy per unit weight (E) is defined as in equation (1) and its time derivative in equation (2) gives a one state dynamic model which can be used to obtain expressions for minimum time and fuel climbs and descents. In equation (1) and those to follow, h is altitude (ft.), V is current velocity (ft/s), D is total drag , T is thrust (lb.), W is the current weight, and f is the current fuel flow rate (lb./s). In the minimum time case, the optimal solution (equation (3a)) is to maximize energy gain per unit time while for the minimum fuel case (equation (3b)) the resulting solution is to maximize energy increase per unit fuel burn. In this approach, near level flight is assumed and rapid variations in the control which may occur are assumed instantaneous (no fuel or time used). Note that equations (3a) and (3b) give optimal climb profiles for gaining energy, and should not be used to construct optimal profiles to a specified range.

$$E = h + \frac{V^2}{2g} \tag{1}$$

$$\dot{E} = V(T - D)/W \tag{2}$$

$$V = \arg\max_V \left(\frac{(T-D)V}{W} \right)_{\substack{T=T_{\max} \\ E=E_{current}}} \tag{3a}$$

$$V = \arg\max_{T,V} \left(\frac{(T-D)V/W}{f} \right)_{E=E_{current}} \tag{3b}$$

The use of SP theory extends the ESA in that it permits more general non-linear dynamic models by taking advantage of multiple time scales which exist in flight mechanics problems. The so-called slow variables are treated in the reduced problem where the fast states are neglected, taking the role of control variables. This is done by introducing a perturbation parameter which identifies the multiple time scale nature of the dynamics. The fast variables are treated in what is termed the boundary layer (or inner) problem, formed by introducing a stretched time scale more appropriate for these variables. A result of this separation is that the endpoint conditions (say between climb and cruise) are matched, and the reduced solution combined with the boundary layer solution should provide a good approximation to the full order results. In long range trajectories, climbs and descents are viewed as boundary layer transitions in energy occurring near the initial and final times. The combined use of SP and the energy can eliminate difficult to solve TPBVP from the solution approach by limiting the number of states considered in the boundary layer and reduced problems. A key objective in using SP is to approximate the optimal open loop control with a near-optimal solution in feedback form.

The second climb solution in FLOPS is for minimum time and fuel climbs/descents based on specified end conditions on the position and energy states (using a 3 state model). Reference 9 derives the solution to the resulting optimal control problems (the fuel and time problems must be developed separately). The solution process used there is complex. In the following few lines, it can be seen how the same results can be achieved via SP formulation. Following the development in reference 11, the problem is approached as follows. The performance criterion given in equation (4) combines a minimum fuel and minimum time formulation, where the σ parameter weights the importance of time and fuel. Thus, the CLIMB random variable in Table 1 represents the σ parameter.

$$J = \int [\sigma f(h,V,T) + (1-\sigma)]dt \tag{4}$$

The point mass dynamics in the vertical plane are used and represented as:

$$\begin{aligned} \dot{x} &= V\cos\gamma \\ \varepsilon\dot{E} &= V(T-D)/W \\ \varepsilon^2\dot{h} &= V\sin\gamma \\ \varepsilon^3\dot{\gamma} &= (L - W\cos\gamma)/mV \end{aligned} \tag{5}$$

where x is the position state, h is altitude, and γ is flight path. The perturbation parameter ε is introduced to facilitate a multiple time scale analysis, in this particular case showing position dynamics to be slower than energy, energy slower than altitude, and altitude slower than flight path angle dynamics. By considering an expansion about ε=0, energy and altitude become control variables in the zeroth-order outer problem and are used to find the optimal cruise point.

Investigating the reduced problem (ε=0), equation (5) and the associated Hamiltonian for the problem give the necessary conditions for optimality shown in equation (6).

$$\begin{gathered} H^o = \lambda_{x_o} V + \sigma f(D,V,h) + (1-\sigma) + \text{constraints} = 0 \\ \frac{\partial H^o}{\partial h} = 0\ ,\ \ \frac{\partial H^o}{\partial E} = 0\ ;\ \ T_o = D,\ L_o = W,\ \gamma_o = 0 \end{gathered} \tag{6}$$

The state variable is x and the controls are now the E and h. The costate λ_{X_o} is found to be constant from

$$\frac{\partial H^o}{\partial x} = \dot{\lambda}_{X_o};\quad \lambda_{X_o} = -(\sigma f + 1 - \sigma)/V \tag{7}$$

and is easily obtainable with $E=E_o$ and $V=V_o$. The formulation in (6) is constrained by altitude and throttle constraints. Analysis of the zeroth-order reduced problem results in the maximization in (8), where f represents fuel flow and the solution h_o, E_o defines the optimal cruise point. The zero subscript indicates these parameters are associated with the outer solution. For σ=1 (min fuel), the optimization translates to maximizing the specific range. For this study, this is the solution adopted for the cruise point solution. In other words, it is only in the climb solution where the σ parameter is introduced as an uncertainty.

$$h_o, E_o = \arg\max_{h,E}\left(\frac{V}{\sigma f(D,V,h)+(1-\sigma)}\right)_{\substack{L=W\\ \gamma=0\\ T=D}} \quad (8)$$

The climb to cruise was obtained by examining the first boundary layer system with E as the state variable and altitude as the control. The necessary conditions for optimality during climb are given in equation (9), resulting in the optimal climb solution of equation (10) for constant energy levels up to E_o.[15] The term λ_{Xo} (evaluated in equation (7)) is the costate from the outer solution and was found to be constant. Knowledge of this costate is not required to construct the optimal reduced state solution. However, as is seen in equation (9), λ_{Xo} does appear in the boundary layer analysis. Thus, the maximization in (10) is performed at each energy level with current values of T, D, V and f and the precomputed value of the costate. The weight dynamics are modeled by calculating the fuel burn during each climb step, and thus updating the weight during the climb procedure. Finally, in the limit as E approaches E_o, the solution in equation (10) approaches the optimal cruise solution in (8).

$$\begin{aligned} H^o &= \lambda_{x_o} V + \lambda_{E_1}(T-D)V/W + \\ &\quad \sigma f(T,V,h) + (1-\sigma) + \text{constraints} = 0 \quad (9) \end{aligned}$$

$$\frac{\partial \mathrm{H}^1}{\partial \mathrm{h}} = 0\ , \quad \frac{\partial \mathrm{H}^1}{\partial \mathrm{T}} = 0\ , \qquad \mathrm{L} = \mathrm{W_o}\ , \ \gamma_o = 0$$

$$h_1, T_1 = \arg\max_{h,T}\left\{\frac{(T-D)V}{\lambda_{X_o} V + \sigma f + (1-\sigma)}\right\}_{\substack{T>D\\ E=E_{current}}} \quad (10)$$

Note that for minimum time problems, thrust appears linearly in the Hamiltonian and thus the climb will occur at full throttle (and descent at min throttle). Comparison of these results with those in Schultz show that the similar conclusions are reached, though the SP result is arrived at in a much more direct way. So far, an assumption of constant and small flight path angle has been made. The investigation of the next layers (i.e. faster flight path angle and altitude dynamics, see equation (5)) is possible, whereas the standard optimal control formulation would encounter difficulties due to the increasing problem size. This is important when the assumptions of instantaneous changes energy allowed by the ESA become non-negligible. For the current case of a transport, though, these dynamics and can be ignored.

Implementation of these SP results and their extensions are currently being pursued. However, for this paper, the existing ESA approach in FLOPS will be used in the interest of demonstrating the RDS technique applied to a problem with mission uncertainty.

Robust Design Simulation

Given these guidance laws, attention now turns towards conducting the analyses/experiments involved in the HSCT design problem. The RDS is an evolving approach, as described in detail reference 7. Key to the approach is the achievement of customer satisfaction by maximizing the probability that the final objective will meet or exceed a given target. As mentioned in the introduction of this paper, the DOE/RSM technique is utilized within RDS in three ways. First, the previously mentioned aerodynamic RSEs are incorporated into FLOPS. Second, RSEs at the system level relate the important elements examined here, including aerodynamic, mission, sizing, and economic parameters. Further, these system level RSEs facilitate the use of the Monte Carlo simulation to account for the effects of the random variables on the solution. Other means are in development by several researches to account for this randomness without resorting to the computationally demanding Monte Carlo method. The Fast Probability Integration (FPI) technique is one promising solution under consideration.[16] For now, however, Monte Carlo will be used, and thus the RSEs are necessary (since running the required thousands of cases of the FLOPS code would be impractical). Finally, RSEs which relate the control parameter settings to probability distributions for the objective and constraints are constructed. These distributions will be used to search a region of the design space for the most robust solution.

A face-centered Central Composite Design (CCD) DOE is selected for the purpose of generating an RSEs relating the responses of interest to the design variables listed in Table I. This particular set-up requires 531 simulations to generate the regression data. The NORMAN program is used in this setting to automate the case definition, execution, and data collection.[17] The RSEs formed here are second order polynomial equations, including main effects, quadratic effects, and interactions between the variables. Once the regression on the DOE data is complete and the equations obtained, both inspection and verification of their statistical properties are required.

Three measures of the validity of the regression are outlined here, though the reader is referred to References 18 through 20 for a more complete treatment on the formation and evaluation of response equations. The R-square value is the square of the correlation between the actual and predicted response.

Figure 3: Prediction Profiles for Intermediate Objectives and Selected Constraints

These prediction profiles, or sensitivities, indicate the behavior of the responses with respect to a change in the design variable settings. The statistical analysis tool used here (*JMP*®)[21] translates a change in the settings with a real time update of the response values (made possible by the underlying simple polynomial equations), giving the designer a feel for the magnitude of the sensitivities. Recall that the perturbation of one design variable causes changes in the responses resulting from main effects (e.g. *X1*), quadratic effects (e.g. *X1*X1*), second order interactions (e.g. *X1*Y1*), and certain third order interactions (e.g. *X1*Y1*Sref*). Economic and weight responses are normalized against the baseline HSCT vehicle in lieu of actual values.

Identifying regions of good designs first facilitates the eventual search for the most robust solutions[22]. If this is not done, the probability of achieving reasonable targets for both the $/RPM and the constraints was found to decrease substantially. In effect, trying to account for unrealistic designs as well as realistic ones (regardless of the uncertainty levels) shifted the total response distributions away from the feasible domain. Also, the regression of probability data can be difficult due to the discontinuous nature of the data (reaching hard limits at zero and one).

Figure 3 also illustrates the use of "desirabilities" for the objectives and constraints, used to perform this required initial search of the design space for regions of good designs. For example, the diagonal desirability shown for $/RPM indicates that its lowest possible value is the most desirable (desirability of one)[7,21]. The constraints have their boundaries marked by a discontinuity. For example, all approach speed values above 154 knots have desirability of zero while all values below 154 knots have desirability of one. One can adjust the design variables according to the desirabilities below them to quickly and interactively get near the optimum. For more rigorous search results, *JMP*® can search the entire design space based on the RSEs and the given desirabilities to find the optimal settings. For the present case, the most desirable settings are shown in Figure 3. The effect of the climb optimization parameter is seen to be small, due to the fact that differences in gross weight between aircraft sized with minimum fuel and time climbs (using the current FLOPS climb routines) are small compared to other effects.

Based on the results of the desirability search, modified ranges for the control and random variables were selected. These are shown in Table IV. Note from Figure 3 that some variables have conflicting trends depending on the response. For example, higher wing reference area decreases $/RPM but increases approach speed. In these cases, the entire range is kept for further analysis.

Table IV: Variable Ranges for Region of Good Designs- Optimization Space

Symbol	*Type*	*Minimum*	*Maximum*
X1	Control	1.54	1.60
Y1	Control	0.52	0.58
Sref	Control	8500 sq. ft.	9500 sq. ft.
TWR	Control	0.28	0.30
DESRNG	Control	5000 nm	5500 nm
SUBL1	Random	0 %	10 %
SUBL2	Random	0 %	10 %
CLIMB	Random	0 (min time)	1 (min fuel)
COFL	Random	0.55 $/gal	1.0 $/gal
EcRNG	Random	3500 nm	5000 nm

If all ten variables were free for the designer to choose, a constrained optimization could be conducted at this point. In fact, the prediction profiles themselves are amenable to "real time" optimization by sliding the bars for each variable and seeing its effect on the objectives and constraints. This is a valuable capability for the purposes of gaining insight and also checking for inconsistent trends in the response models. However, since the variables *SUBL1, SUBL2, COFL, CLIMB,* and, *EcRNG* are not controllable parameters, they must be assigned probability distributions, chosen to reflect any available knowledge about the nature of their variability. When there is a lack of knowledge about this nature, triangular distributions are selected around a midpoint value. Figure 4 on the following page depicts the distribution assumptions used for this study.

Next, a new DOE is constructed, this time only for the control variables (*X1, Y1, TWR, Sref, DESRNG*), to define the "control variable" design space. Again a CCD experiment is chosen, this time with five variables examined at five levels resulting in 30 simulations which need to be run. In the previous application of DOE/RSM, FLOPS was the analysis tool used to generate the responses of $/RPM, weights, and the constraints. These RSEs become inputs to a Monte Carlo simulation, which is the analysis tool used to generate the new responses. These new responses are probability distributions for $/RPM, gross weight, and constraints. The experiment table defines the 30 combinations (settings of variables) to be analyzed via the simulation subject to the assumptions in Figure 4. Each results in a frequency plot of the simulation data. To ensure a statistically meaningful result, 5,000 cases were executed during the Monte Carlo simulation.

Figure 5: Sample Frequency Chart and Fitted Probability Distribution for $/RPM from Monte Carlo

RDS Results

The *constraints* as well as the objectives are affected by the uncertainty. Thus, they too will result in probability distributions. This phenomenon is due to the fact that the mission uncertainty affects the performance of the aircraft (unlike economic uncertainty, which does not). Now, to find a robust solution, the settings of control variables which maximize the probability of meeting a target value for $/RPM while *simultaneously* achieving a probability of one (or very close to one) of meeting the constraints need to be determined. Recall that the constraint values are defined in Table II. The desirability search procedure is utilized again by assigning a desirability of one to high probabilities of meeting three different targets for $/RPM. This is seen in Figure 6, showing the prediction profiles. Each response depicted represents a regression equation which relates the control parameters to a probability distribution for a given target. This is constrained by meeting the approach speed constraint for any possible combination of uncertainties. Thus, a probability of one of achieving a 154 knot approach speed is desired. Economic results are again normalized by the baseline.

Three targets are specified for the $/RPM in Figure 6. Target A is the most aggressive since it represents the lowest $/RPM value of the three, followed in ascending order by B and C. Naturally, the probability of achieving the target increases from target A to C. Comparing against the intermediate (or traditional) desirability results of Figure 4, the robust search results show similar trends for three of the five control variables (*X1*, *TWR*, and *DESRNG*). The normalized y-kink location (*Y1*) takes on an intermediate value in the robust approach, and the wing area variable (*SREF*) should be set at its midpoint for the most robust feasible solution. The latter effect seems to be due to the conflicting goals of maximizing the probability of meeting the approach speed constraint and maximizing the achievement of low $/RPM targets. The other performance constraints tracked, the takeoff and landing field lengths, both resulted in probabilities of one for *every* point tested in the region of good designs. In other words, these constraints are not active.

Figure 6: Prediction Profiles for Objective and Constraint Probabilities

Table V and Figure 7 summarize the robust solution results as compared to the baseline HSCT. The table lists the five control variable settings associated with the robust solution as well as their baseline values.

Table V: HSCT Robust Solution Results

Control Variable	***Baseline***	***Robust Solution***
X1	1.615	1.55
Y1	.51	.565
TWR	.3	.28
Sref	8500 sq. ft.	9000 sq. ft.
DESRNG	5000 nm	5000 nm

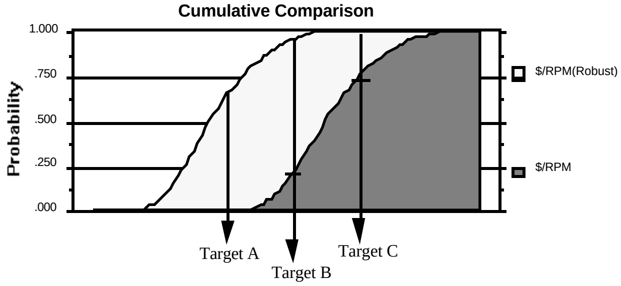

Figure 7: Cumulative Distribution Comparison

The power of the RDS method, though, is seen best in a comparison of the cumulative distributions of the baseline and robust aircraft (Figure 7). For each target, the robust solution has a significantly higher probability of achieving the target. In fact, for the lowest target (target A), the baseline had a probability of zero. This graphic summarizes in a concise way the result of a RDS implementation. This result completes the objective stated at the outset of the paper of determing a link between mission uncertainty, optimal guidance, wing planform, and economic parameters and using that linkage to arrive at a robust HSCT design.

Future Work

The identification of good designs first, regardless of the uncertainty levels, makes the probability RSEs easier to construct. It does seem, though, that the relative width of the economic uncertainty variables exceeded that of the mission and climb random variables. Further, investigations into whether this importance difference is a true result or is due to a ranging problem is warranted. Finally, implementation of SP guidance algorithms in the process is also of importance, perhaps investigating the use of the climb optimization factor as a control in the formulation.

Conclusions

A method has been explored which makes use of both analytical and statistical approximation methods in conducting a Robust Design Simulation (RDS), recognizing that aircraft design is not a deterministic process. Existing climb optimization algorithms in a synthesis code were shown to be derivable via reduced order modeling and Singular Perturbation (SP) techniques, and prospects for extending these existing algorithms using SP modeling seem attractive. To demonstrate the RDS, existing climb methods in the synthesis and sizing code FLOPS are used in a design optimization of a High Speed Civil Transport in the presence of mission and economic uncertainties as well as performance constraints. Through intermediate response surface equations (RSEs), a Monte Carlo simulation was conducted to evaluate the effect of modeled uncertainty. This resulted in new RSEs representing probabilities of meeting a series of objective targets as a function of wing planform and vehicle sizing variables. Further, it was found that the *constraints* are uncertain as well, resulting in distributions for them. These probabilities were then used to determine the robust solution. An attractive aspect of the entire approach is that comparison of alternative guidance solutions or uncertainty models across a spectrum of missions and aircraft configurations is now more realizable.

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References

1. Mavris, D. N., Bandte, O., Schrage D.P., "Economic Uncertainty Assessment of an HSCT using a Combined Design of Experiments / Monte Carlo Approach," 17th Annual Conference of the ISPA, San Diego, CA, May 1995.

2. Mavris, D. N., Bandte, O., Schrage D.P., "Effect of Mission Requirements on the Economic Robustness of an HSCT," 18th Annual Conference of the ISPA, Cannes, France, June, 1996.

3. Schrage, D. P., "Concurrent Design: A Case Study", Chapter 21 from Concurrent Engineering, Automation, Tools, and Techniques by Andrew Kusiak, A Wiley-IntersciencePublication, John Wiley & Sons, Inc., New York, New York, 1993.

4. Maciejowski, J.M., Multivariable Feedback Design, Addison-Wesley Publishing, 1989.

5. Ray, L. R., Stengel, R.F., “Stochastic Measures of Performance Robustness in Aircraft Control Systems”, *Journal of Guidance, Control, and Dynamics*, Vol. 15, No. 6, Nov.-Dec., 1992.

6. Chamis, C., “Probabilistic Methods for Structural Design and Reliability”, 8th SAE Reliability, Maintainability, Supportability & Logistics Conference, Dallas, TX, May, 1996.

7. Mavris, D.N., Bandte, O., “Application of Probabilistic Methods for the Determination of an Economically Robust HSCT Configuration”, AIAA/USAF/NASA/ISSMO Multidisciplinary Analysis and Optimization Conference, Bellevue, WA, September 4-6, 1996.

8. McCullers, L.A. *Flight Optimization System* User’s Guide, Version 5.7, NASA Langley Research Center, 1995.

9. Schultz, R.L., Zagalsky, N.R., “Aircraft Performance Optimization”, *Journal of Aircraft*, Vol.9, No.2, February 1972.

10. Bryson, A.E., Desai, M.N., Hoffman, W.C., “Energy-State Approximation in Performance Optimization of Supersonic Aircraft”, *Journal of Aircraft*, Vol.6, No.6, November-December 1969.

11. Calise, A.J., “Extended Energy Management methods for Flight Performance Optimization”, *AIAA Journal*, Vol. 15, pp. 314-321, 1977.

12. Calise, A.J., “Singular Perturbations in Flight Mechanics”, (book chapter) in Applied Mathematics in Aerospace Science and Engineering, Plenum Press, New York, 1994.

13. DeLaurentis, D.A., Mavris, D.N., Schrage, D.P., “An IPPD Approach to the Preliminary Design Optimization of an HSCT using Design of Experiments”, to be presented at 20th ICAS Congress, Sorrento, Italy, 8-13 September 1996.

14. Galloway, T.L., and Mavris, D.N., *Aircraft Life Cycle Cost Analysis (ALCCA) Program*, NASA Ames Research Center, September 1993.

15. Aggarwal, R., Calise, A.J., Goldstein, F., "Singular Perturbation Analysis of Optimal Flight Profiles for Transport Aircraft", JACC, San Francisco, CA, June 1977.

16. Wirsching, P. H., Wu, Y. T., "Advanced Reliability Methods for Structural Evaluation", *Transaction of the ASME*, Vol. 109, February 1987.

17. *NORMAN*- A Simulation Experiment Sequencing System, IMEC, Leuven Belgium, 1995.

18. Box, G.E.P., Draper, N.R., Empirical Model-Building and Response Surfaces, John Wiley & Sons, Inc., New York; 1987.

19. Box, G.E.P., Hunter, W.G., Hunter, J.S., Statistics for Experimenters, John Wiley & Sons, Inc., New York, 1978.

20. Montgomery, D.C., Design and Analysis of Experiments, John Wiley & Sons, Inc., New York, 1991.

21. SAS Institute Inc., *JMP Computer Program and User's* Manual, Cary, NC, 1994.

22. Chen, W., Allen, J.K., Mavris, D.N., Mistree, F., Tsui, K-L, 1995b, "Integration of Response Surface Method with the Compromise Decision Support Problem in Developing a General Robust Design Procedure," *Advances in Design Automation* (Azarm, S., et al. Eds.), New York: ASME, 1995, pp. 485-492. ASME DE-Vol. 82-2.

show min fuel, time climbs for the robust solution

Once identified, it will facilitate the creation of the much needed methodology for including optimal guidance in the MDO problem

- SP versus approximation RSE for inner loop control
- Set up Cliff: SP, Point mass. linearization
- Set up RSE: RB, linearized, RSE

While a point mass modeling of EOM was sufficient for obtain guidance laws for performance evaluation, inner loop control investigation will need more detailed models. Guidance Laws feed control commands to the flight control system. Along with probable stability augmentation duties, the flight control system must be able to have the authority to achieve such commands. The guidance law commands themselves, as well as the resulting FCS requirements, then, will be influenced by choices the designer might have made or will make in terms of planform geometry, engine selection, engine /location, etc.

For the designer, these types of issues are important in the conceptual and preliminary stages. An "optimized" configuration which cannot meet the FAR 36 Noise limits or satisfy minimum handling qualities criteria is not a feasible design. Thus, for an HSCT example, an ability to investigate near-optimal trajectories with noise constraints active or to investigate the benefits of a statically unstable configuration may indicate design drivers previously unforeseen in the problem. The designer can then address these issues. TRIM